Guidelines for Interactive Reliability-Based Structural Optimization using Quasi-Newton Algorithms

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STRUCTURAL RELIABILITY THEORY
PAPER NO. 156

Presented at the 7th IFIP WG7.5 Working Conference on Reliability and Optimization of Structural Systems, Boulder, Colorado, USA, April 2-4, 1996

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Guidelines for Interactive Reliability-Based Structural Optimization using Quasi-Newton Algorithms

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Abstract: Guidelines for interactive reliability-based structural optimization problems are outlined in terms of modifications of standard quasi-Newton algorithms. The proposed modifications minimize the condition number of the approximate Hessian matrix in each iteration, restrict the relative and absolute increase of the condition number and preserve positive definiteness without discarding previously obtained information. All proposed modifications are also valid for non-interactive optimization problems.
Heuristic rules from various optimization problems concerning when and how to impose interactions such as fix/relax design variables, include/exclude constraints, etc. are considered and visualized through a realistic structural example.

Keywords: Interactive Optimization, Quasi-Newton Algorithms, Update of Hessian Matrix, Condition Number, Reliability-Based Optimization.

1 INTRODUCTION

In this paper interactive reliability-based structural optimization problems are considered. During the optimization process the designer is able to modify simple bounds, fix or relax design variables, include or exclude constraints, adjust parameters concerning the reliability analysis and the optimization algorithm, etc. Only quasi-Newton optimization algorithms and reliability analyses using FORM (First Order Reliability Method) are considered.

From a simple perturbation analysis of a linear matrix system, see e.g. Gill et al. [4], the sensitivity of the solution is shown to be proportional to the condition number of the matrix, i.e. the ratio between the maximum and minimum eigenvalue of the matrix. Based on this observation, it is proposed to perform interactive optimization based on a modified quasi-Newton algorithm, which includes a subproblem that minimizes the condition number of the approximate Hessian matrix in each iteration and thereby stabilizes the determination of the search direction in each iteration after major interactions. Additionally, algorithm dependent guidelines that effectively reduce obsolete and unnecessary evaluations of functions and gradients are considered.

In short, the purpose is to formulate and outline guidelines and heuristic rules for when and how to interact and, concurrently, modify existing algorithms to comply with the interactive modifications imposed by the designer. Still, the proposed modifications are required to apply to traditional non-interactive optimization problems.

2 FORMULATION OF RELIABILITY-BASED OPTIMIZATION PROBLEM

The general formulation of a non-linear inequality constrained optimization problem is

$$\min_{\mathbf{z}} \quad C(\mathbf{z}) \tag{1}$$

$$c_j(\mathbf{z}) \geq 0 \qquad j = 1, ..., m_i \tag{2}$$

$$z_i^{\min} \leq z_i \leq z_i^{\max} \qquad i = 1, ..., n \tag{3}$$

where $\mathbf{z}^T = (z_1, z_2, ..., z_n)$ denotes the design variables, $C(\mathbf{z})$ is the objective function and $c_j(\mathbf{z})$ the constraints. Introducing the Lagrange multipliers λ_i, the Lagrange function $L(\cdot)$ is defined by (4), where the $2n$ simple bounds for brevity are included as inequality constraints of the standard form (2), i.e.

$$L(\mathbf{z}, \boldsymbol{\lambda}) \;=\; C(\mathbf{z}) - \textstyle\sum_{j=1}^{m_i} \lambda_j c_j(\mathbf{z}) - \sum_{i=1}^{2n} \lambda_{m_i+i} c_i(\mathbf{z}) \tag{4}$$

The reliability-based optimization problem is obtained when a subset of the constraints (2) is formulated in terms of the reliability index $\beta_j(\mathbf{z})$ and the minimum acceptable reliability index $\beta_j^{\min}$, i.e. $c_j(\mathbf{z}) = \beta_j(\mathbf{z}) - \beta_j^{\min}$. In this paper, the probability of failure $P_f = \Phi^{-1}(\beta_j)$ is approximated by the FORM solution (see e.g. Thoft-Christensen & Baker [14] or Madsen et al. [8]), i.e. $\beta_j(\mathbf{z})$ is found iteratively in a normalized space $\mathbf{U}$.

3 QUASI-NEWTON OPTIMIZATION ALGORITHM

Using a quasi-Newton algorithm, optimum $\mathbf{z}^*$ is found iteratively as the limit of the sequence $\{\mathbf{z}^{(k)}, k = 1, 2, ...\}$. The iterate $\mathbf{z}^{(k+1)}$ is obtained as $\mathbf{z}^{(k+1)} = \mathbf{z}^{(k)} + \alpha\mathbf{d}$, where $\mathbf{d}$ denotes the search direction while the step length α is obtained from a one-dimensional line search in the direction $\mathbf{d}$ in a merit function $\psi(\cdot)$. In this paper, the socalled $L1$-function proposed by Han [6] is used as merit function, i.e.

$$\psi(\mathbf{z}) = C(\mathbf{z}) + \textstyle\sum_{j=1}^{m_i} \mu_j \mid \min\left[0, c_j(\mathbf{z})\right] \mid \tag{5}$$

where μ_j is penalty parameters. The termination criterion for the line search is based on the relaxed criterion known as the watchdog technique proposed by Chamberlain et al. [3].

The search direction $\mathbf{d}$ is in each iteration determined as the solution to a sequential quadratic programming (SQP) subproblem of the form

$$\min_{\mathbf{d}} \quad \nabla_z C(\mathbf{z}^{(k)})^T \mathbf{d} + \frac{1}{2}\mathbf{d}^T \mathbf{B}^{(k)} \mathbf{d} \tag{6}$$

$$\text{s.t.} \quad c_j(\mathbf{z}^{(k)}) + \nabla_z c_j(\mathbf{z}^{(k)})^T \mathbf{d} \geq 0 \qquad j = 1, ..., m_a \tag{7}$$

$$z_i^{(k)} - z_i^{\min} \leq d_i \leq z_i^{\max} - z_i^{(k)} \qquad i = 1, ..., n \tag{8}$$

where only a subset of all m_i inequality constraints in (2) is included in the linearized constraints (7). $\mathbf{B}^{(k)}$ denotes a symmetric and positive definite matrix which gradually is updated by use of a quasi-Newton scheme (see section 5) to approach the Hessian matrix of the Lagrange function $L(\cdot)$.

Introducing a column vector $\mathbf{h}(\mathbf{z})$ in which the active inequality constraints and active simple bounds are assembled, the solution of (6)-(8) is obtained from the linear system

$$\begin{bmatrix} \mathbf{B}^{(k)} & -\nabla_z \mathbf{h}(\mathbf{z}^{(k)}) \\ -\nabla_z \mathbf{h}(\mathbf{z}^{(k)})^T & 0 \end{bmatrix} \begin{bmatrix} \mathbf{d} \\ \boldsymbol{\kappa}^{(k+1)} \end{bmatrix} = \begin{bmatrix} -\nabla_z C(\mathbf{z}^{(k)}) \\ \mathbf{h}(\mathbf{z}^{(k)}) \end{bmatrix} \tag{9}$$

where the Lagrange multipliers $\boldsymbol{\kappa}^{(k+1)}$ correspond to the QP subproblem (6)-(8). Succeedingly, the estimate of $\boldsymbol{\lambda}$ to the original problem (1)-(3) is updated as $\boldsymbol{\lambda}^{(k+1)} = \boldsymbol{\lambda}^{(k)} + \alpha(\boldsymbol{\kappa}^{(k+1)} - \boldsymbol{\lambda}^{(k)})$. Standard solution techniques which iteratively determines the set of constraints in $\mathbf{h}(\mathbf{z})$ are outlined in Ringertz [12] and Golub & Van Loan [5]. General algorithm dependent information such as a modification ensuring that the linearized constraints on $\mathbf{d}$ in (7) are consistent can be found in Powell [10] and Schittkowski [13].

4 USER IMPOSED INTERACTIONS

In this paper, the changes that the designer are able to modify interactively are

- Fix and relax design variables.
- Modify current value of design variables.
- Modify simple bounds, include or exclude constraints.

A qualitative discussion of desirable interactive changes and information charts such as what-if studies, sensitivities, etc. upon which the modifications are based can be found in Arora [1] and Arora & Tseng [2].

Considering the interaction that a design variable is fixed to a user specified value, a straightforward strategy is to reduce the dimension of the subproblem (6)-(8). However, change of the dimension of the Hessian matrix $\mathbf{B}^{(k)}$ results in loss of already obtained information in cases where temporarily fixed design variables are re-included in the optimization problem. Additionally, the rows and columns that correspond to the re-included variable z_i must fulfil certain requirements in order to preserve positive definiteness of $\mathbf{B}^{(k)}$ which is an important property to obtain stability of the SQP since the 2nd term $\frac{1}{2}\mathbf{d}^T\mathbf{B}\mathbf{d}$ in (6) otherwise is unbounded.

An alternative approach is to include an additional equality constraint per fixed design variable in (6)-(8) whereby the full dimension of $\mathbf{B}^{(k)}$ and positive definiteness is preserved. Thus, from the modified solution scheme (10) for the search direction subproblem in the case where design variables z_i is fixed, it is easily seen that d_i is equal to zero.

$$\begin{bmatrix} \mathbf{B}^{(k)} & -\nabla_z \mathbf{h}(\mathbf{z}^{(k)}) & \mathbf{e}_i \\ -\nabla_z \mathbf{h}(\mathbf{z}^{(k)})^T & \mathbf{0} & \mathbf{0} \\ \mathbf{e}_i^T & \mathbf{0} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{d} \\ \boldsymbol{\kappa}^{(k+1)} \end{bmatrix} = \begin{bmatrix} -\nabla_z C(\mathbf{z}^{(k)}) \\ \mathbf{h}(\mathbf{z}^{(k)}) \\ 0 \end{bmatrix} \tag{10}$$

In (10) the column vector $\mathbf{e}_i = \{0, ...0, 1, 0, ...0\}^T$ contains zeroes except at the ith position. Due to the presence of the additional constraint, the corresponding active constraint set $\mathbf{h}(\mathbf{z}^{(k)})$ is generally different from the active set that corresponds to the standard problem (9) where all design variables are allowed to vary. Finally, the last element in the vector $\boldsymbol{\kappa}^{(k+1)}$ is seen to correspond to the artificial constraint $d_i = 0$.

Since $d_i = 0$, it is seen from (10) that recalculation of the derivative $\partial C/\partial z_i$ and $\partial h_j/\partial z_i$ for the fixed z_i can be omitted without affecting $\mathbf{d}$. Still, the values of $\boldsymbol{\kappa}^{(k+1)}$ are dependent whereby it is suggested to reuse the lastest evaluated derivative $\partial C/\partial z_i$ and $\partial h_j/\partial z_i$ where z_i was active in the gradients $\nabla_z C(\mathbf{z}^{(k)})$ and $\nabla_z \mathbf{h}(\mathbf{z}^{(k)})$ in (10).

With respect to the interactive inclusion/exclusion of constraints, change of current design point and simple bounds, the major difficulties originate from the fact that the Hessian matrix $\mathbf{B}^{(k)}$ is depending on these changes. Therefore, how to update $\mathbf{B}^{(k)}$ is considered in the next section.

5 UPDATE OF THE HESSIAN MATRIX

The revision of the approximate Hessian matrix $\mathbf{B}^{(k)}$ must satisfy the quasi-Newton condition - see e.g. Gill et al. [4]

$$\mathbf{B}^{(k+1)}\mathbf{p}^{(k)} = \mathbf{q}^{(k)} \tag{11}$$

where $\mathbf{p}^{(k)}$ is the difference between the current and previous design point while $\mathbf{q}^{(k)}$ is dependent upon the difference between the current and previous derivatives of the Lagrangian $L(\mathbf{z}, \boldsymbol{\lambda})$. A frequently used definition of $\mathbf{q}^{(k)}$ is (13) proposed by Powell [10], where only the most recent set of Lagrange multipliers $\boldsymbol{\lambda}^{(k)}$ is used and the additional relaxation parameter $\theta_k \in [0,1]$ is introduced in order to preserve positive definiteness.

$$\begin{aligned}
\mathbf{p}^{(k)} &= \mathbf{z}^{(k)} - \mathbf{z}^{(k-1)} = \alpha \mathbf{d}^{(k)} && (12)\\
\mathbf{q}^{(k)} &= \theta_k(\nabla_z L(\mathbf{z}^{(k)}, \boldsymbol{\lambda}^{(k)}) - \nabla_z L(\mathbf{z}^{(k-1)}, \boldsymbol{\lambda}^{(k)}) + (1-\theta_k)\mathbf{B}^{(k)}\mathbf{p}^{(k)} && (13)
\end{aligned}$$

Having defined $\mathbf{p}^{(k)}$ and $\mathbf{q}^{(k)}$, $\mathbf{B}^{(k)}$ is usually revised using a symmetric rank-two update formula. In accordance with Gill et al. [4] and Luenberger [7] the one-parameter Broyden family update formula may be written as

$$\mathbf{B}^{(k+1)} = \mathbf{B}^{(k)} - \frac{\mathbf{B}^{(k)}\mathbf{p}^{(k)}\mathbf{p}^{(k)^T}\mathbf{B}^{(k)}}{\mathbf{p}^{(k)^T}\mathbf{B}^{(k)}\mathbf{p}^{(k)}} + \frac{\mathbf{q}^{(k)}\mathbf{q}^{(k)^T}}{\mathbf{q}^{(k)^T}\mathbf{p}^{(k)}} + \phi_k \left(\mathbf{p}^{(k)^T}\mathbf{B}^{(k)}\mathbf{p}^{(k)}\right)\mathbf{w}\mathbf{w}^T \tag{14}$$

where the vector $\mathbf{w}$ is defined as $\mathbf{w} = \mathbf{q}^{(k)}/(\mathbf{q}^{(k)^T}\mathbf{p}^{(k)}) - \mathbf{B}^{(k)}\mathbf{p}^{(k)}/(\mathbf{p}^{(k)^T}\mathbf{B}^{(k)}\mathbf{p}^{(k)})$.

Choosing $\phi_k = 1$ the formula (14) is termed the DFP (Davidon-Fletcher-Powell) update while $\phi_k = 0$ results in the efficient and even more commonly used BFGS (Broyden-Fletcher-Goldfarb-Shanno) update formula - e.g. in NLPQL by Schittkowski [13].

In order to minimize the condition number of the Hessian, a straightforward choice of $\mathbf{B}^{(k+1)}$ is obtained from the following additional subproblem to be solved in each iteration

$$\begin{aligned}
\min_{\phi_k} \quad & \text{cond}(\mathbf{B}^{(k+1)}) = f(\mathbf{B}^{(k)}, \mathbf{p}^{(k)}, \mathbf{q}^{(k)}, \phi_k) && (15)\\
\text{s.t.} \quad & \lambda_1(\mathbf{B}^{(k+1)}) = \lambda_{\min} > 0 && (16)
\end{aligned}$$

where a positive lowest eigenvalue (16) ensures positive definiteness of $\mathbf{B}^{(k+1)}$.

In (15)-(16), the relaxation parameter θ_k introduced in (13) is given by the self-scaling scheme by Powell [10]. Having obtained a fixed value for the parameter ϕ_k from (15)-(16), a 2nd subproblem is introduced for the purpose of reducing the influence of the relaxation term $\mathbf{B}^{(k)}\mathbf{p}^{(k)}$ in (13) subject to constraints that prescribe an upper absolute limit (K_1) and maximum relative increase (K_2) in the condition number of $\mathbf{B}^{(k+1)}$, i.e.

$$\begin{aligned}
\min_{\theta_k} \quad & (1-\theta_k) && && (17)\\
\text{s.t.} \quad & \lambda_1(\mathbf{B}^{(k+1)}) = \lambda_{\min} > 0 && && (18)\\
& \text{cond}(\mathbf{B}^{(k+1)}) < K_1 && , \text{ say } K_1 = \min(10^{3+\sqrt{n}}, 10^8) && (19)\\
& \text{cond}(\mathbf{B}^{(k+1)})/\text{cond}(\mathbf{B}^{(k)}) < K_2 && , \text{ say } K_2 = \max(n^2, 10^2) && (20)\\
& 0 \le \theta_k \le 1 && && (21)
\end{aligned}$$

In the limit $\theta_k = 0$, it is seen from (13)-(14) that the Broyden update (14) results in the trivial result $\mathbf{B}^{(k+1)} = \mathbf{B}^{(k)}$. Thus, provided that $\text{cond}(\mathbf{B}^{(k)}) < K_1$, a feasible solution with respect to the constraints (19)-(20) exists.

A more detailed description of the Hessian update is given in Pedersen [9]. Among other methods, a technique can be used where old and obsolete information in the approximate Hessian is discarded by use of $\mathbf{p}^{(k)}$ and $\mathbf{q}^{(k)}$ vectors from the most recent iterations only. Using this approach, the Hessian matrix is first initialized in each iteration after which (14) is applied successively, say $0.5n$ times.

6 GUIDELINES FOR INTERACTIONS

Based on the experiences from several optimization problems varying from simple polynomials to realistic deterministic and reliability-based structural problems with $n \in [2, 15]$ and $m_i \in [1, 20]$, the following tentative guidelines for interactive changes and adjacent modifications of the quasi-Newton algorithm have shown generally applicable:

Guidelines for Interactive Changes Imposed by the Designer

- Change of the current design point (typically combined with change in simple bounds with the intention to obtain a new optimal design that is more acceptable) is done when the active set of constraints has stabilized and the norm of the search direction vector $\| \mathbf{d} \|$ is decreasing.
- Introduction of fixed design variables is most effective after $\mathbf{z}$ has stabilized and the value of the fixed ith design variable is close to the previous value $z_i^{(k)}$.
- All design variables with relatively large influence/sensitivity with respect to an active and/or violated constraint (e.g. adjoint diameter/thickness of a given member) should not be fixed at the same time.
- Interactive change of the current design point from a feasible to a substantially violated point must be omitted. What-if studies and other qualitative assessments of the changes are recommended in order to verify that a solution exists in this neighbourhood and that the next optimum is mostly found from the feasible region.
- Inclusion of constraints, hereby also narrowing of the simple bounds, must be done with respect to feasiblity.

Algorithm Dependent Guidelines

- No automatic initialization of the Hessian matrix $\mathbf{B}$ is performed if the active set of constraints $\mathbf{h}(\mathbf{z})$ in (9) is constant (or almost constant) and the step length $\alpha = 1$. (In many standard algorithms, $\mathbf{B}$ is typically initialized every nth iteration to avoid ill-conditioning)
- Initialize $\mathbf{B}$ if the line search in the merit function $\phi(\cdot)$ not is fulfilled for an extremely small value of the step length α, say $\alpha \approx 10^{-6}$.
- In cases where $\mathbf{p}^T\mathbf{B}\mathbf{p} \gg \mathbf{p}^T\mathbf{p}$ for a well-scaled problem, the Hessian matrix $\mathbf{B}$ is typically ill-condtioned. The 2nd order term $\frac{1}{2}\mathbf{d}^T\mathbf{B}\mathbf{d}$ in (6) completely dominates the the linear term $\nabla_z C(\mathbf{z})^T\mathbf{d}$.

- Depending on the degree of non-linearity, cond($\mathbf{B}$) greater than, say $10^{3+\sqrt{n}}$ causes ill-conditioning and poor search directions after change in current design point.
- Since the computational effort used to evaluate the gradients of the reliability-based constraints is small once $\beta(\mathbf{z})$ and thereby the function value is found (see section 2), the number of line search must be minimized. Hence, modest values of the penalty parameters μ_j used in the merit function and relaxed stop criteria such as the watchdog technique by Chamberlain [3] must be used.
- Considering evaluation of reliability-based constraints, adaptive accuracy is highly effective, i.e. the termination criterion for the FORM reliability analysis is controlled by the value of $c_j(\mathbf{z})$ - refer e.g. to Pedersen [9]. Hereby the number of obsolete limit state evaluations is minimized.

7 EXAMPLE - OFFSHORE JACKET

In order to illustrate the effects of the guidelines outlined in section 6 and the modifications of the algorithm, a reliability-based optimization of an 'academic' offshore jacket is considered. The geometry of the 48-bar truss structure is shown in figure 1.

The algorithm is implemented into a toolbox called IROS (Interactive Reliability-based Optimization System) to be used in the MATLAB environment - see Pedersen [9] for a more detailed description. An open environment is hereby provided for the designer, where a large series of built-in functions concerning numerical analyses, matrix computations and graphical facilities are available. The user defined problem is specified in standardized files, upon which function and gradient evaluations are performed by an external reliability analysis programme (Reliab01 [11]) and an external FEM programme through a standardized interface. A large variety of structural problems is hereby solved by IROS.

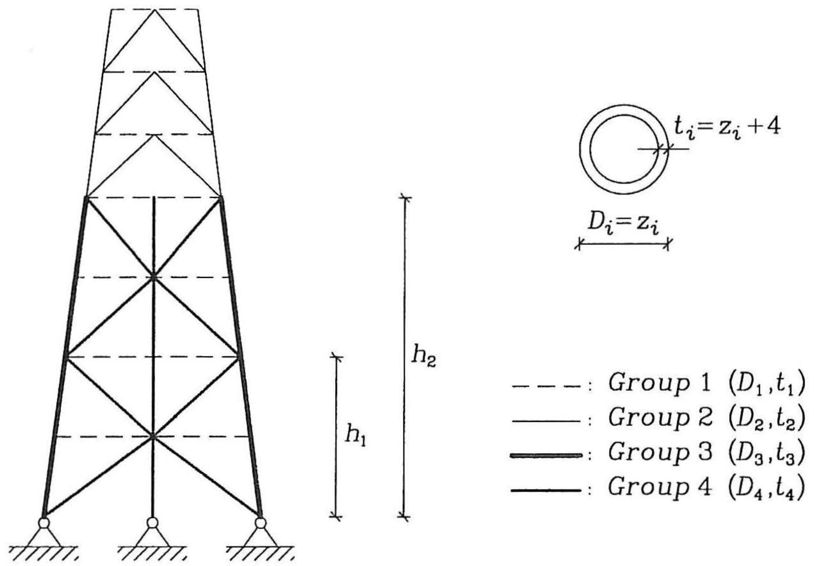

Figure 1: Overall geometry of offshore jacket and definition of element groups.

The optimization problem applies to the standard form (1)-(3), where the volume of the truss is used as objective function. The reliability-based constraints consider maximum deflection, yielding and instability, while a deterministic constraint ensures $h_1 < h_2$. In order to simplify the response model and minimize the computational effort used to evaluate limit state functions, only extreme short term loads and the most critical elements in each of the 4 elements groups are considered. Design variables $\mathbf{z}$, constraints $\mathbf{c}$ and stochastic variables $\mathbf{x}$ are tabulated in table 1.

Design Variables					
Variable	Designation		Lower	Initial	Upper
$z_1 \dots z_4$	Diameter of cross-sec. [1,2,3,4]	[mm]	500.0	var.	2500.0
$z_5 \dots z_8$	Thickness of cross-sec. [1,2,3,4]	[mm]	10.0	var.	100.0
z_9, z_{10}	Vertical coordinate [1,2]	[m]	0	var.	175.0
Constraints					
Const.	Designation		Type	$\beta^{\min}$	Limit
$c_1 \dots c_6$	Yielding in cross-sec. [1,2,3,4]		Rel.	4.0	f_y
$c_7 \dots c_{12}$	Global instability in cross-sec. [1,2,3,4]		Rel.	4.0	-
$c_{13} \dots c_{18}$	Local instability in cross-sec. [1,2,3,4]		Rel.	4.0	-
c_{19}	Horizontal deflection of top site		Rel.	2.0	$U_{\max}$
c_{20}	Ordering of vertical heights		Det.	-	$h_1 < h_2$
Stochastic Variables					
Variable	Designation		Distrib.	Exp. value	Coef. of Var.
$x_1 = f_y$	Yielding strength	[N/mm^2]	LN	450.0	0.08
$x_2 = E$	Modulus of elasticity	[N/mm^2]	LN	$2 \cdot 10^5$	0.04
$x_3 = C_D$	Drag coefficient	[-]	N	1.2	0.25
$x_4 = t_m$	Marine growth thickness	[m]	LN	0.15	0.50
$x_5 = V$	Wind velocity	[m/s]	EX1	40.0	0.20
$x_6 = H$	Extreme wave height	[m]	EX1	25.0	0.15

Table 1: Design variables, stochastic variables and constraints.

The interactive reliability-based optimization of the 48-bar truss is carried out in accordance with the follwing 4 steps:

1: Initial optimization with wide simple bounds in order to find the 'global' optimum.

2: Change of the current design point and narrowing of the simple bounds of the diameters in order to find a more acceptable solution.

3: Move diameters to discrete integer values and fix those.

4: Re-include diameters, move thicknesses to discrete integer values and fix those.

In figures 2 and 3, the iteration history of the design variables, objective function and minimum value of the constraints (i.e. most violated) in each iteration are plotted for two different strategies:

A: Hessian update where the condition no. is minimized according to (14)-(21) using the watchdog line search termination criterion.

B: Hessian update according the BFGS scheme using a standard line search criterion.

In both cases, the design variables have been scaled to unity before the 1st iteration while the objective function is scaled with respect to $\nabla_z C(\mathbf{z}^{(0)})$. Additionally, no update of the

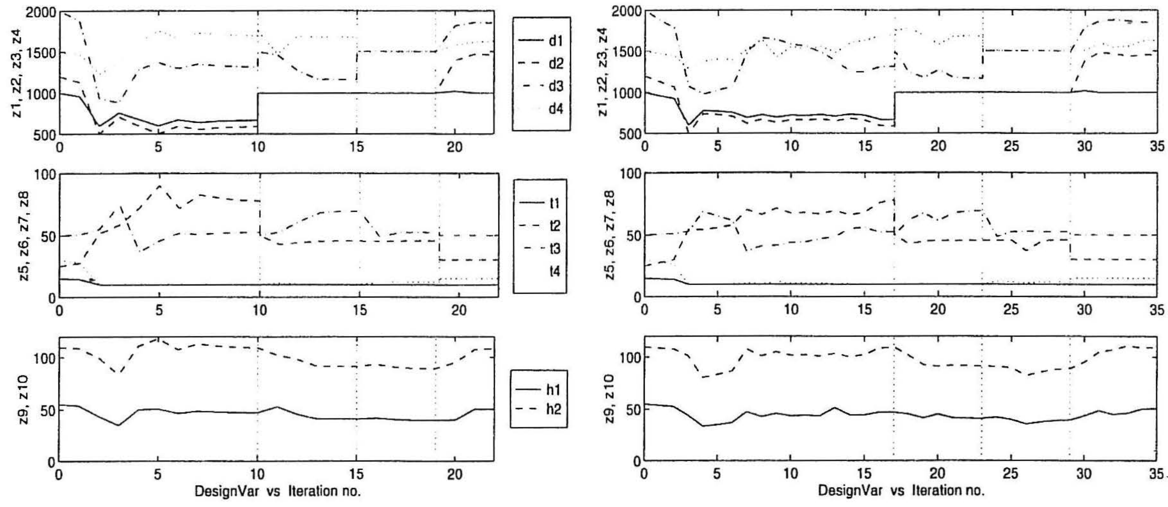

Figure 2: Iteration history for design variables for the two strategies A (left) and B (right).

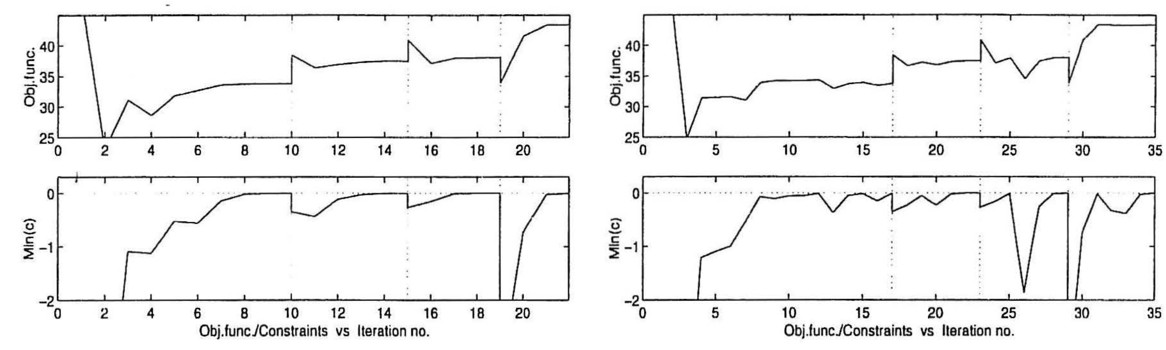

Figure 3: Iteration history for objective function and the minimum value of all constraints for the two strategies A (left) and B (right).

Hessian matrix is performed after an interaction where the current design point $\mathbf{z}$ is moved since the vectors $\mathbf{p}$ and $\mathbf{q}$ in (12)-(13) are biased after this type of interactions.

In order to be able to compare the required no. of iterations in each step for the two strategies, the optimization is continued until mild convergence criteria concerning $\| \mathbf{d} \|$, $\| \nabla_z L(\cdot) \|$ and $\psi(\cdot) - C(\cdot)$ is fulfilled - see also figure 4 for the iteration history of these convergence parameters. The required no. of iterations for each step for strategy A and B and 3 alternative strategies are summarized in table 2.

Optimization strategy				No. of iterations in step				
Type	Hessian update	Initialization	Line search	1	2	3	4	Total
A	min cond($\mathbf{B}$)	Never	Watchdog	10	5	4	3	22
-	BFGS update	$\mathbf{B} = \mathbf{I}$ if cond($\mathbf{B}$) $> 10^{10}$	Watchdog	16	6	5	5	32
-	BFGS update	$\mathbf{B} = \mathbf{I}$ after interac.	Watchdog	16	6	10	10	42
B	BFGS update	$\mathbf{B} = \mathbf{I}$ if cond($\mathbf{B}$) $> 10^{10}$	No watchdog	17	6	6	6	35
-	BFGS update	$\mathbf{B} = \mathbf{I}$ after interac.	No watchdog	17	6	10	10	43

Table 2: Required no. of iterations for 5 different optimization strategies.

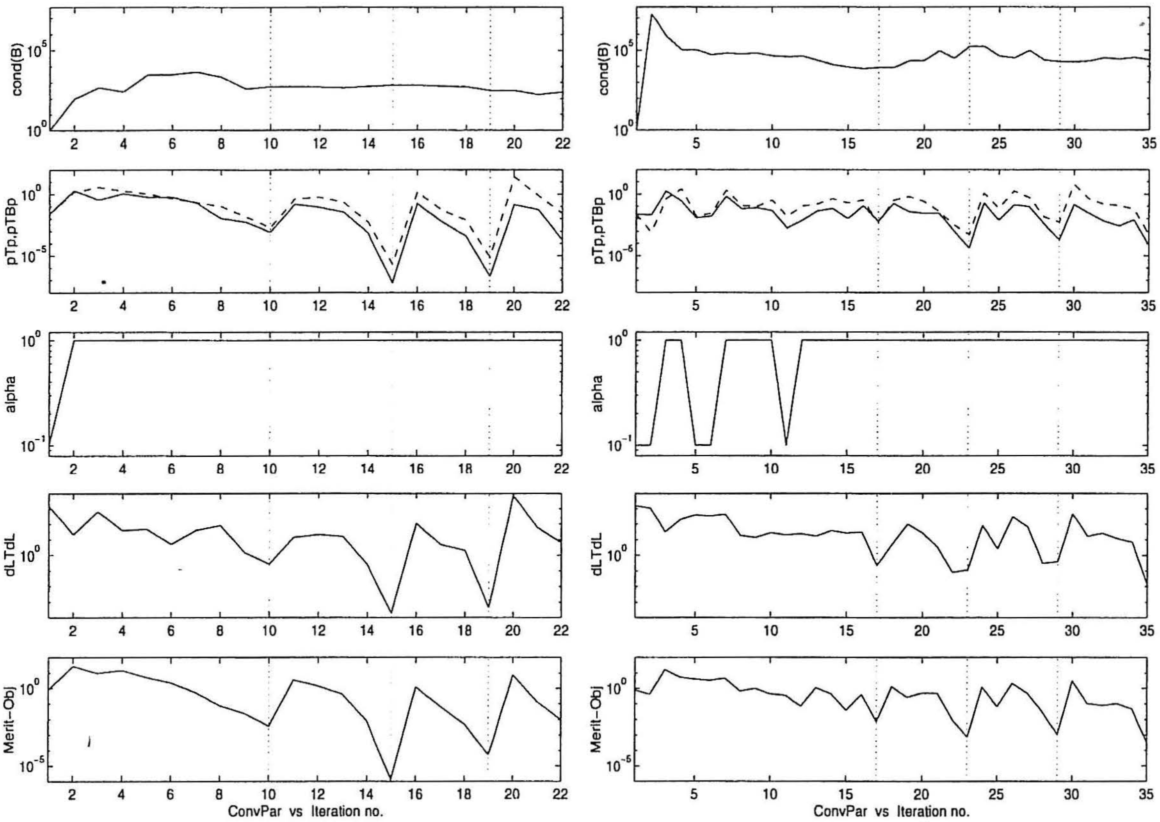

Figure 4: Iteration history of cond($\mathbf{B}$), scalar product of step vector $\mathbf{p}^T\mathbf{p}$ and product $\mathbf{p}^T\mathbf{B}\mathbf{p}$ (dashed), step length in line search α, scalar product of gradient of the Lagrange function (dLTdL) and difference between merit and objective function for the two strategies A (left) and B (right).

Comparing the required no. of iterations after the 1st interaction (i.e. steps 2, 3 and 4), strategy A is seen to converge quicker than the other alternatives. Even more importantly, figures 2-4 show that the transition from one optimum to the next is seen to be smooth and fast when using the min cond($\mathbf{B}$) strategy compared to the more fluctuating curves when using the standard BFGS update.

Considering the 1st step before any interaction is imposed, strategy A requires fewer iterations (10) than strategy B (17). Compared with the results from the alternative optimizations in table 2, this increased no. of iterations is partly due to the line search criterion used in strategy B (step length $\alpha < 1$ as shown in figure 4). In the context of reliability-based optimization, where the computational effort used to evaluate the gradients ($d\beta/dz_j = \mid \nabla_z g(\cdot) \mid^{-1} \partial g(\cdot)/\partial z_j$) is small compared to function evaluations (found iteratively by FORM), the watchdog technique or other modest line search criteria where $\alpha = 1$ is often accepted, are preferable. However, since $\alpha \equiv 1$ after the first interaction after step 1, the difference in the line search criterion used in strategies A and B is insignificant in relation to the comparison of the performance of the methods in steps 2, 3 and 4.

Considering the condition no. of the Hessian matrix for the two strategies which are depicted in figure 4, cond($\mathbf{B}$) for strategy A is seen to be approximately a factor 10^2 –

10^3 smaller than the standard BFGS update. For non-scaled problems (i.e. $z_i/z_j \gg 1$, $\partial C/\partial z_i \gg 1$, $\partial h_j/\partial z_i \gg 1$, etc. for some i, j), the min cond($\mathbf{B}$) strategy has proven even more efficient compared to the standard BFGS since the latter typically entails values of cond($\mathbf{B}$) in the range of $10^8 - 10^{10}$, where numerical problems in the solution of the QP (6)-(8) appear. Furthermore, only a small change of the current design point affects the search direction $\mathbf{d}$, partly by the change of Hessian matrix $\mathbf{B}$ itself and partly by the large sensivity of $\mathbf{B}$. Also, refer to Gill et al. [4] where a perturbation analysis is shown.

In other words, the Hessian update (14)-(21) seems to be an appropriate compromise of how to include 2nd order information without fine-tuning the Hessian matrix to a specific point in the given design space which can be used for both non-interactive and especially interactive reliability-based optimization problems.

8 ACKNOWLEDGEMENT

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